Kalman Filter based Observer Design for Handling Dynamics : The Sideslip Estimation Problem

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Kalman filter background

Popular method !   (174 hits for 'vehicle Kalman filter' - Compendex '95 - '00)
eg vehicle handling/stability, suspension control, global positioning

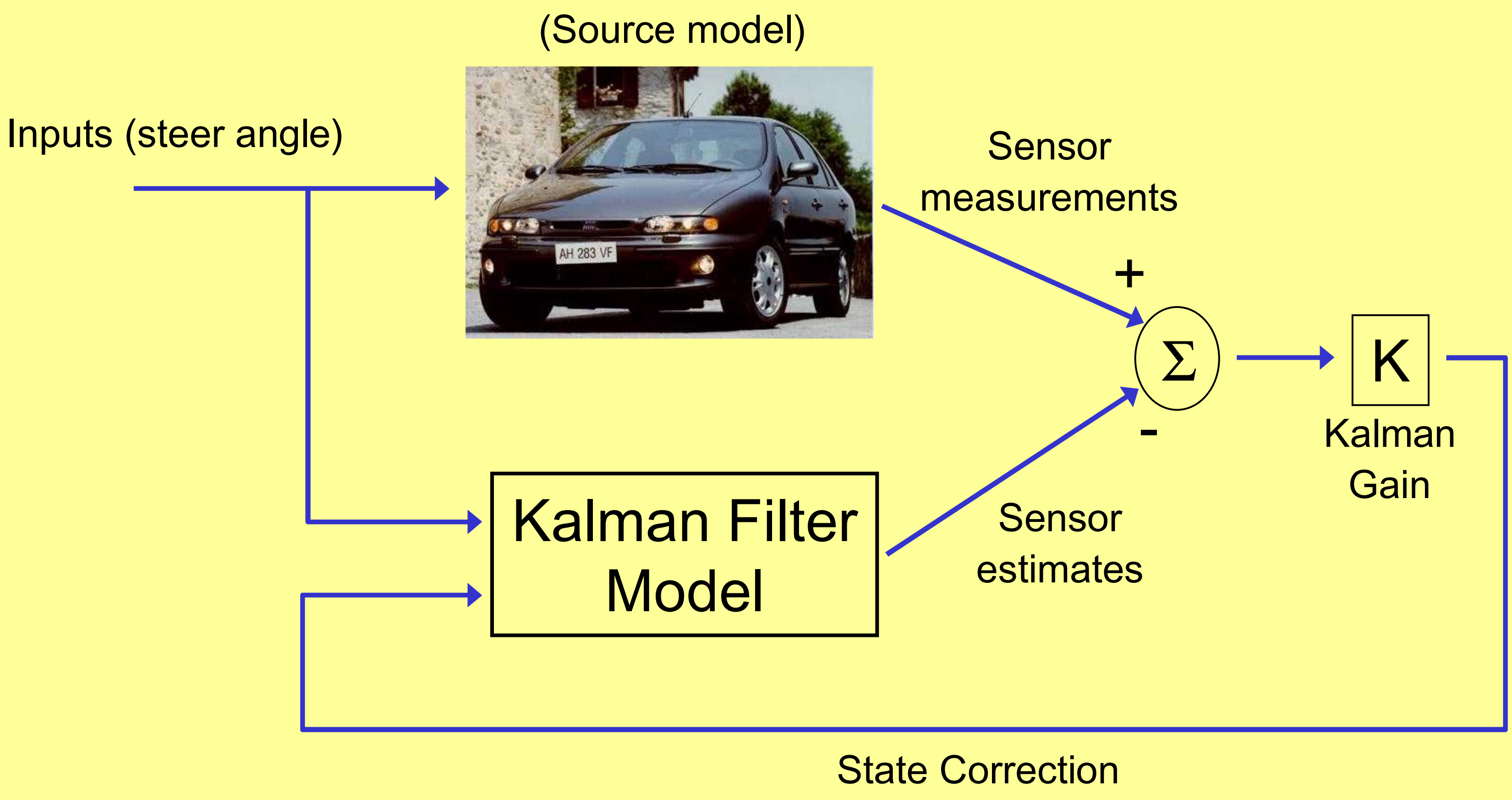
Classical uses :

- Real-time state reconstruction
- Filtering

Specific vehicle handling goals :

- Reconstruction of handling states from small sensor set (real-time ?)
- . . . particularly side-slip velocity (vehicle attitude when cornering)
- Tyre force prediction / modelling
- Model identification

Kalman Filter Operation



The Linear Kalman Filter

$$\boldsymbol{x}(k+1) = \mathbf{A_d}\,\boldsymbol{x}(k) + \mathbf{B_d}\,\boldsymbol{u}(k) + \boldsymbol{w}(k)$$

$$\boldsymbol{y}_s(k) = \mathbf{C}\,\boldsymbol{x}(k) + \mathbf{D}\,\boldsymbol{u}(k) + \boldsymbol{v}(k)$$

$\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$: **true** states and inputs

$\boldsymbol{y}_s(k)$: sensor measurement

$\boldsymbol{w}(k)$: modelling errors

$\boldsymbol{v}(k)$: sensor model errors + measurement noise

For an optimal observer, $\boldsymbol{w}(k)$ and $\boldsymbol{v}(k)$ are *zero mean white noise* processes

$$\mathbf{e}_k = \begin{bmatrix} \boldsymbol{\omega}_k \\ \boldsymbol{\upsilon}_k \end{bmatrix}, \quad E\left(\mathbf{e}_k \mathbf{e}_k^{\mathrm{T}}\right) = \begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^{\mathrm{T}} & \mathbf{R} \end{bmatrix} \longrightarrow \text{'balance' the Kalman Filter (LQR design)} \longrightarrow \boldsymbol{K}$$

$$\boldsymbol{x}_e(k+1) = \mathbf{A_d}\boldsymbol{x}_e(k) + \mathbf{B_d}\boldsymbol{u}(k) + \boldsymbol{K}\,\{\,\boldsymbol{y}_s(k) - \mathbf{C}\boldsymbol{x}_e(k) - \mathbf{D}\boldsymbol{u}(k)\,\}$$

Three Simulation Studies on Handling Observers

Study 1 : Linear and Linear Adaptive Kalman Filter

Best M.C. and Gordon T.J., '*Real-Time State Estimation of Vehicle Handling Dynamics Using an Adaptive Kalman Filter'* proceedings from the 4th International Symposium on Advanced Vehicle Control (AVEC), Nagoya, Japan, September 1998, pp 183-188

Study 2 : Nonlinear Adaptive Kalman Filter

Best M.C., Gordon T.J. and Dixon P.J., *'An Extended Adaptive Kalman Filter for Real-time State Estimation of Vehicle Handling Dynamics,'* Vehicle System Dynamics : International Journal of Vehicle Mechanics and Mobility, Vol 34, No 1, pp 57-75, 2000.

Study 3 : Nonlinear Adaptive for states and parameters

Best M.C. and Gordon T.J., '*Combined State and Parameter Estimation of Vehicle Handling Dynamics'* proceedings from the 5th International Symposium on Advanced Vehicle Control (AVEC), Ann Arbor, USA, August 2000, pp 429-436

Study 1 summary

Source model : Yaw / roll / sideslip with

- Pacejka lateral tyre force model
- lateral load transfer & inclined roll axis
- constant forward speed

KF model : Linear yaw / sideslip, ‘bicycle’

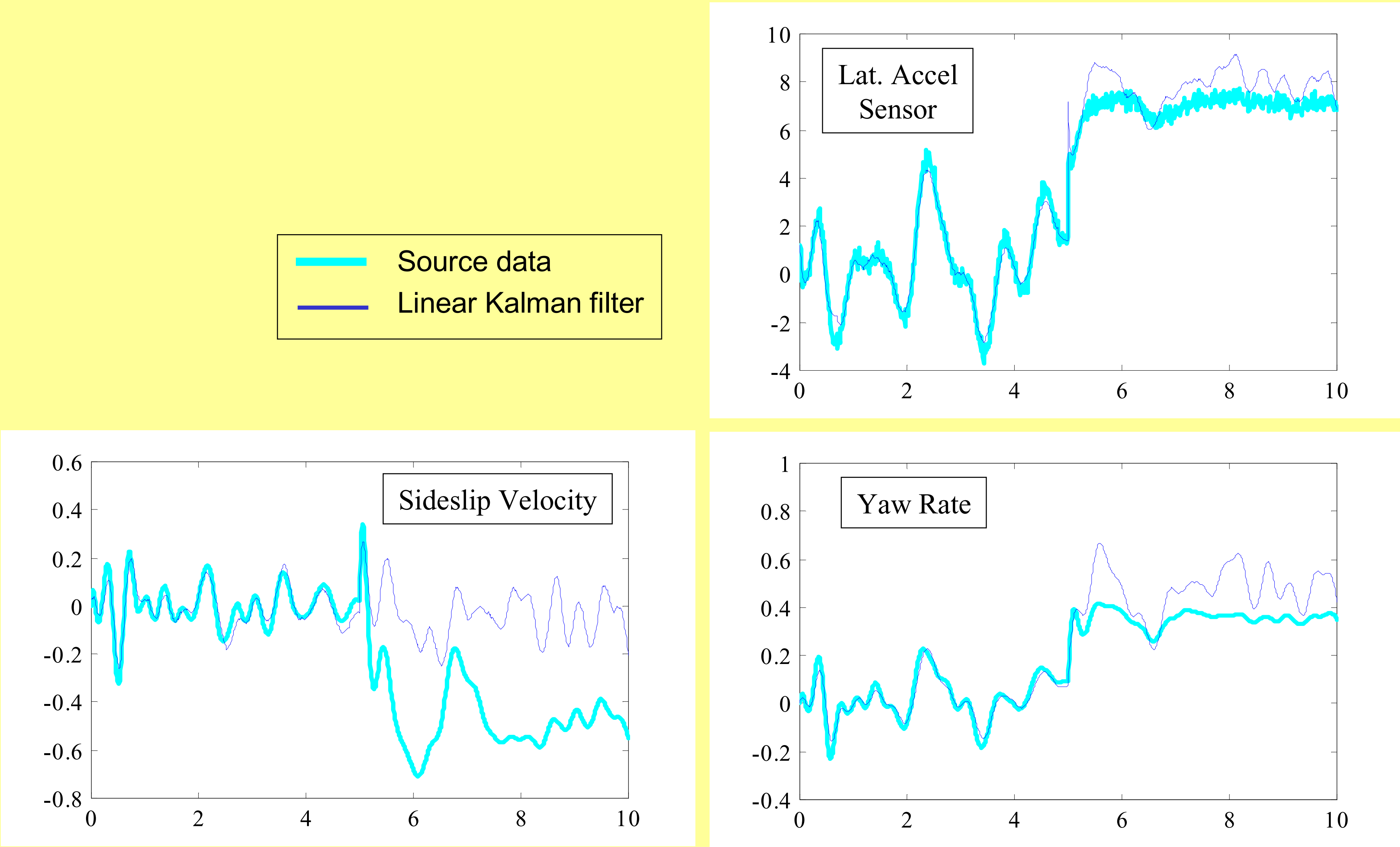
KF algorithm : Linear & Linear time varying

Inputs : Steer angle

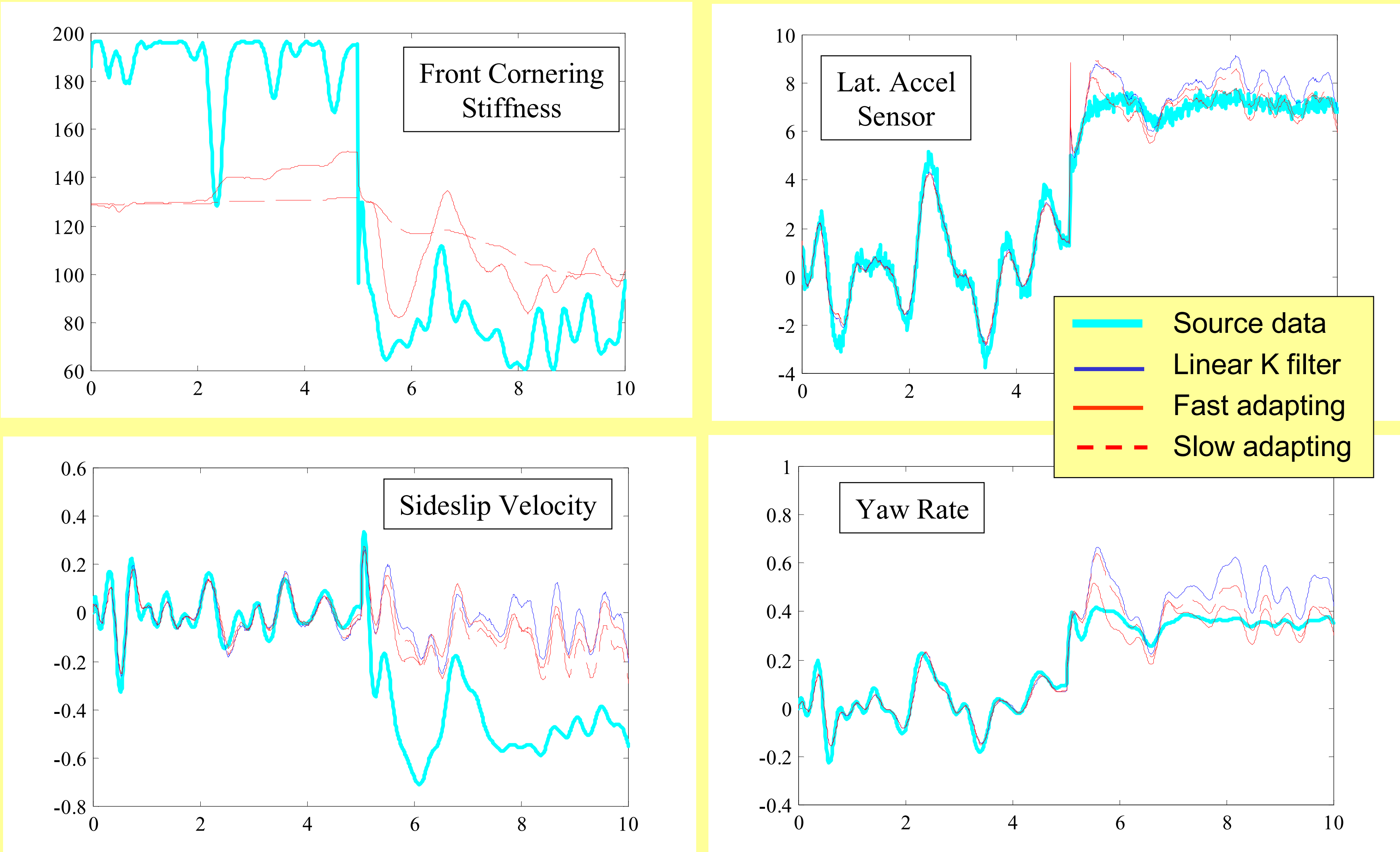
Sensors : One or two Lateral Accels

Adaptation : Recursive least-squares based on estimated states

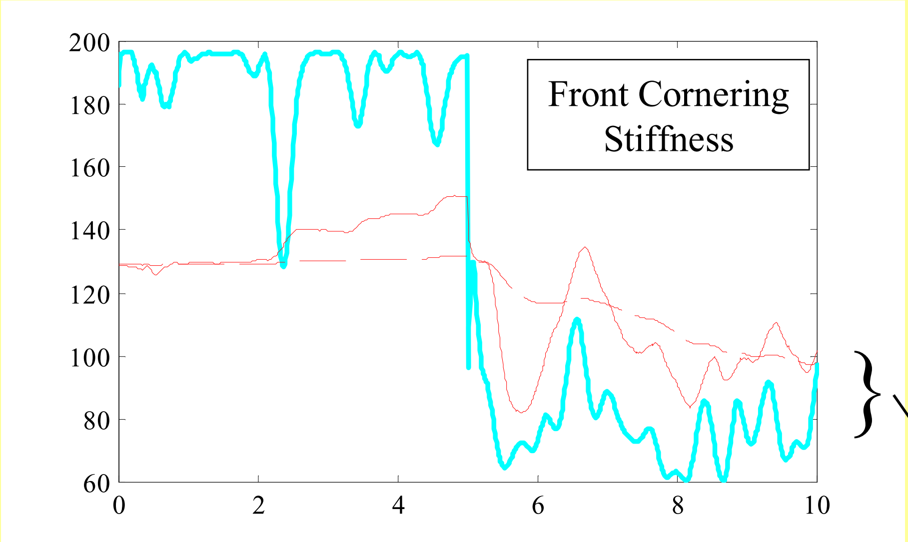
Study 1 results

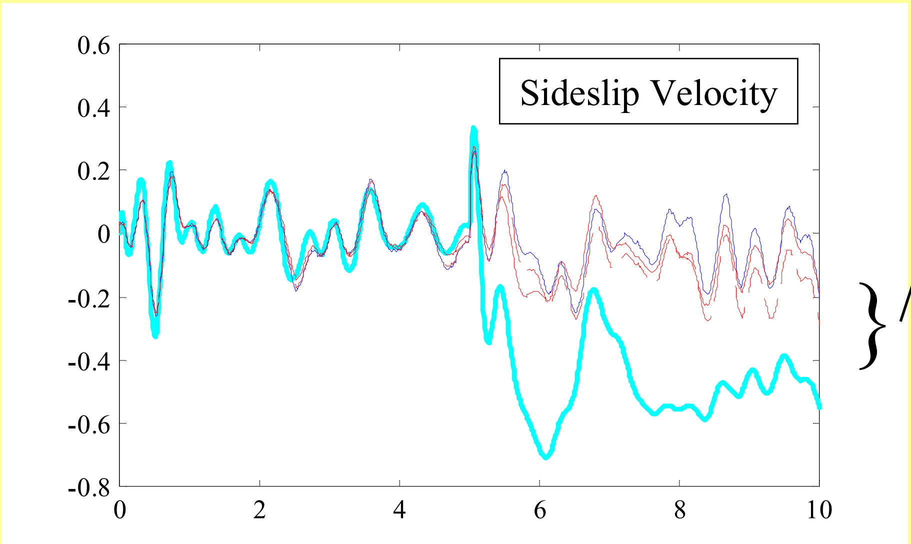


Study 1 results



Study 1 results





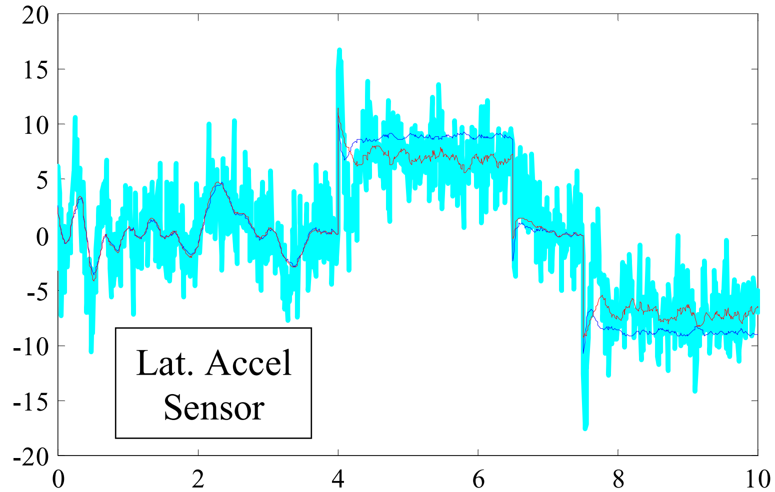
Errors due to under-determined model :

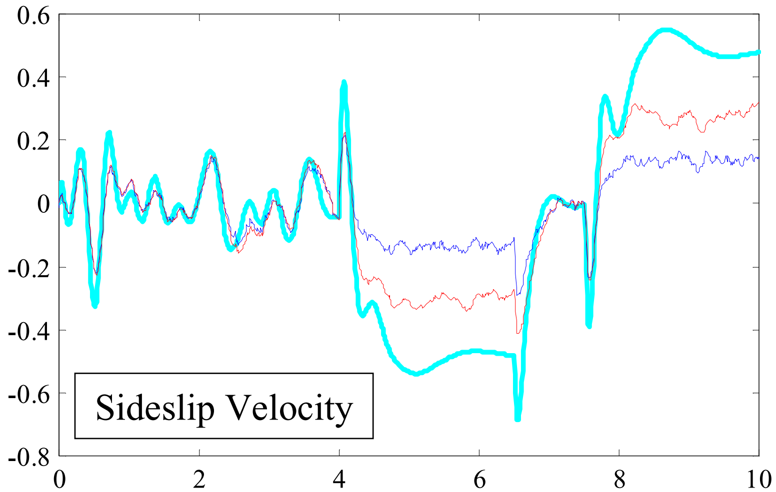
$$\hat{C}_{\alpha f} = C_{\alpha f}\left(\frac{v - cr}{\hat{v} - cr}\right), \qquad \hat{C}_{\alpha r} = C_{\alpha r}\left(\frac{u\delta - v - br}{u\delta - \hat{v} - br}\right)$$

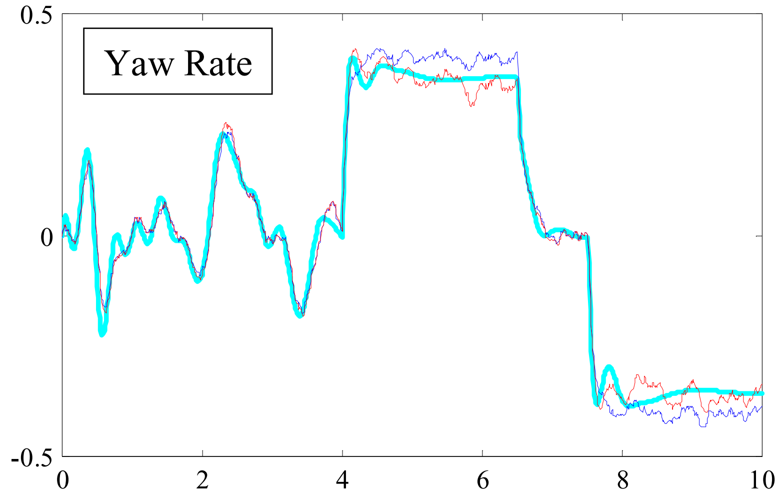
Study 1 results

Performance under high sensor noise condition

Source data
Linear K Filter
Fast adapting







Study 1 Conclusions

Good yaw rate estimation

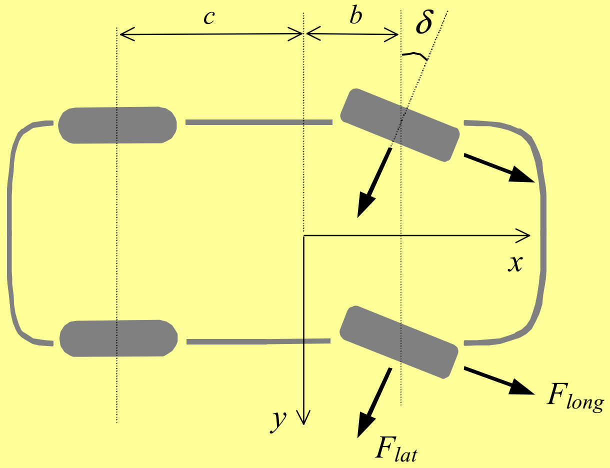
Excellent noise rejection

Model must be more detailed (higher order ?)

ß to avoid steady-state problems with Sideslip Velocity

Study 2 plan :

Introduce longitudinal mode, so steer induced deceleration decouples $C_{\alpha f}$ and v :



$$M\dot{u} = \frac{bC_{\alpha f}\delta}{u}r + \left(\frac{C_{\alpha f}\delta}{u} + Mr\right)v + Mhrp + \mu(w-u) - C_{\alpha f}\delta^2$$

Also introduce roll mode to expand on number of estimated states

Study 2 summary

Source model : Real-time race game model

KF model : Yaw / roll / sideslip / longitudinal

KF algorithm : Extended (nonlinear), involving

- recursive form of Ricatti equation
- matrix inversion
- Jacobian (derivative matrix) calcs

Sensors : Longitudinal + 3 Lateral Accels

Adaptation : Nonlinear algorithm allows parameters to be defined as (eg slow-varying) states :

$$\dot{\boldsymbol{\chi}}(t) = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{C}_{\alpha f}(t) \\ \dot{C}_{\alpha r}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}(t)) \\ \mathbf{f}_{\alpha}(\mathbf{x}(t)) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}(t) \\ \boldsymbol{\omega}_{\alpha}(t) \end{bmatrix}$$

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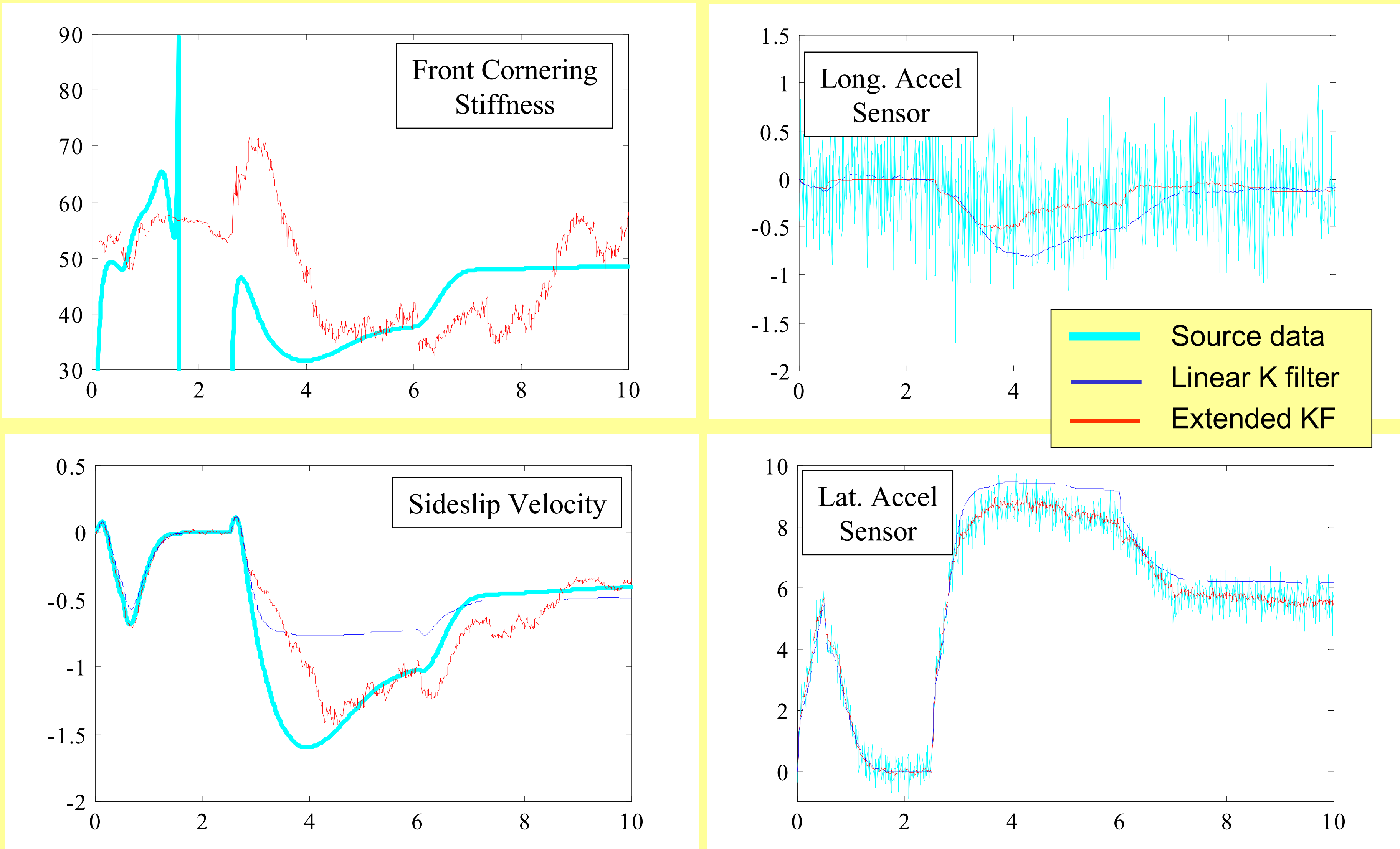
Study 2 results



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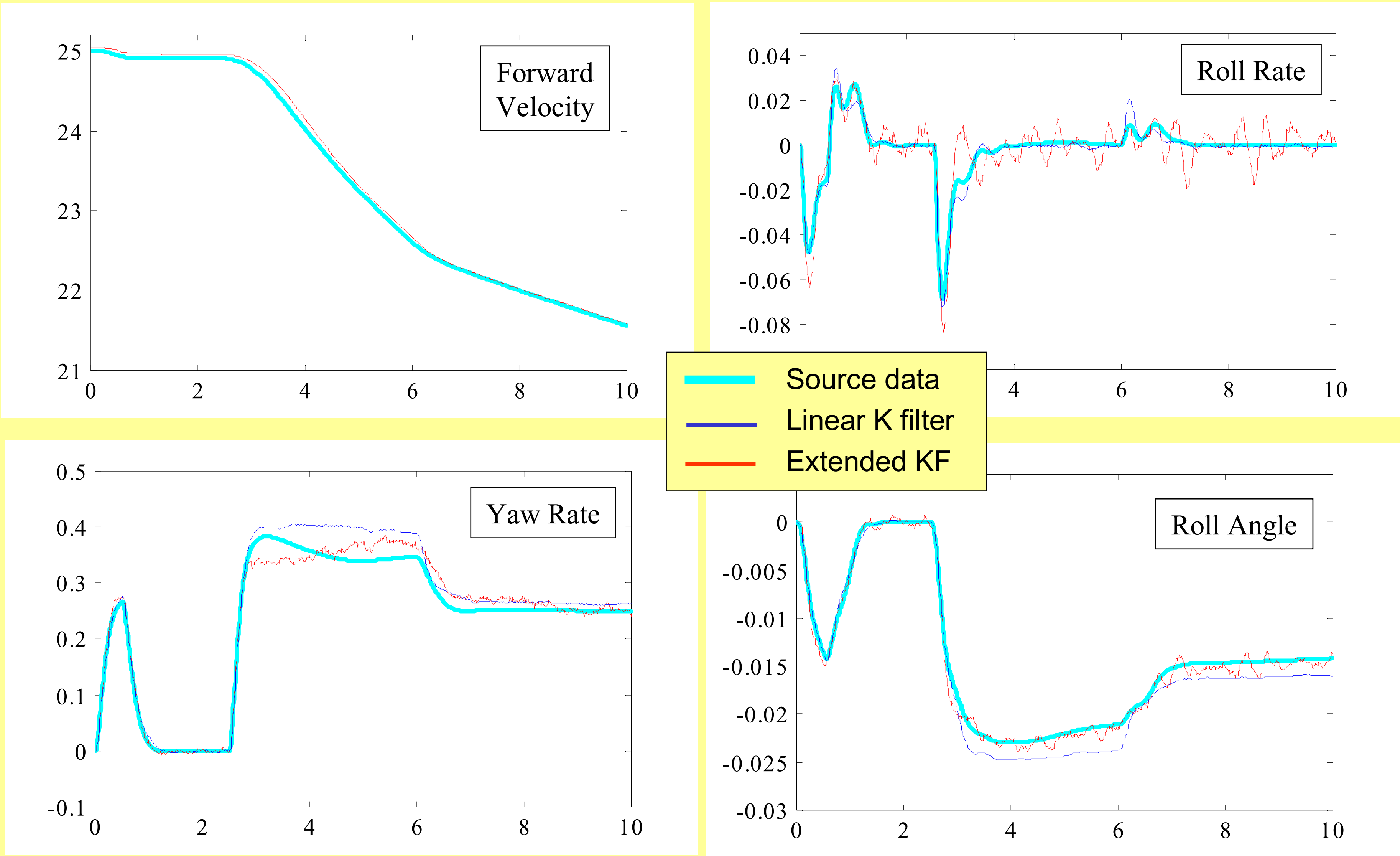
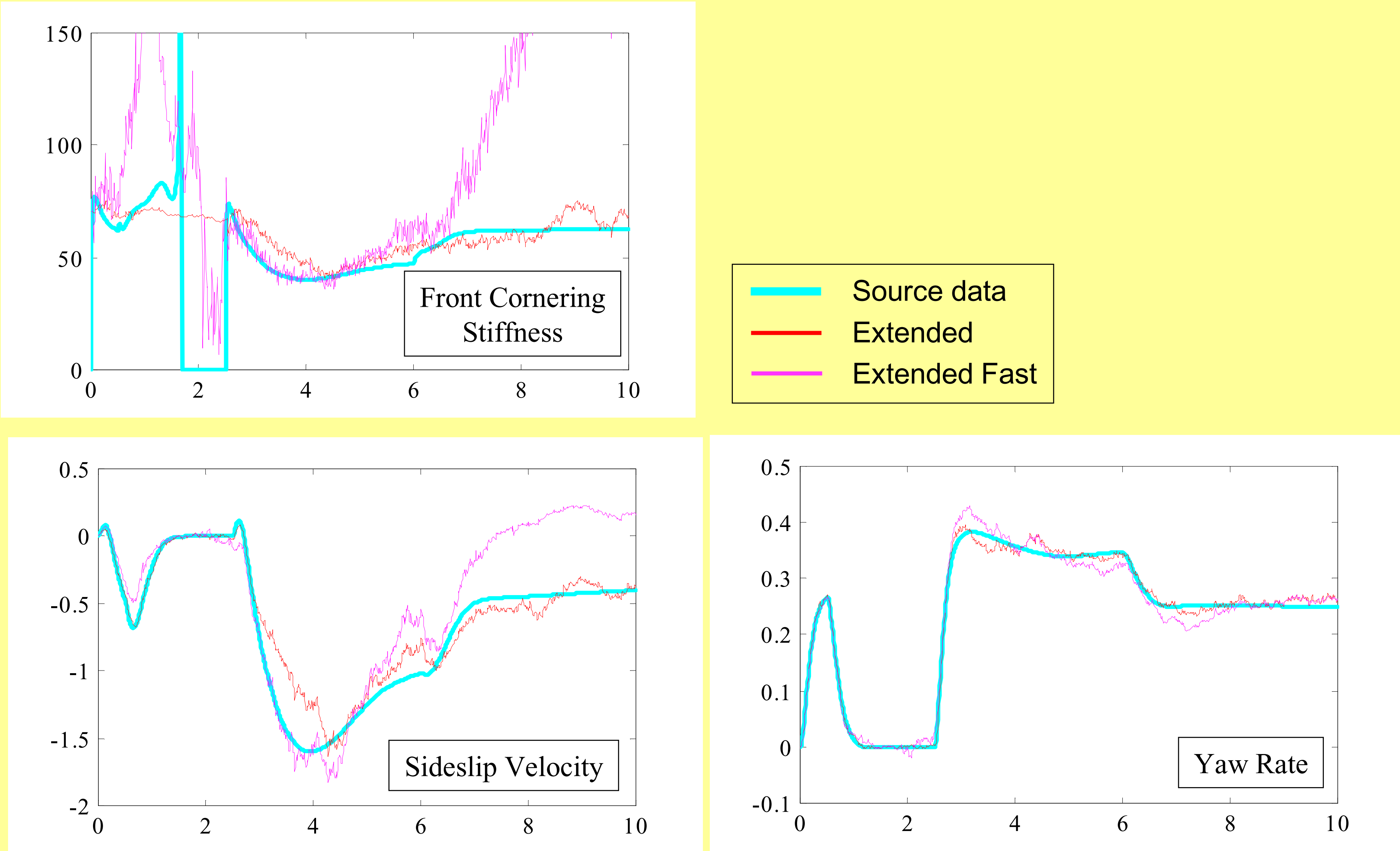
Study 2 results



Study 2 results



Study 2 Conclusions

Basic principle works - better Sideslip estimation

- ß Roll mode rather noisy (noise matrix tuning ?)
- ß Unstable if adaptation rate is too high

? Why ‘re-identify’ tyre with a changing C_α

⟶ Use nonlinear tyre in KF model

Study 3

Similar models and design to study 2, but :

- KF model now includes Pacejka formula for tyre forces
- More emphasis on scope for parameter adaptation (on-line model identification), through :

$$\dot{\boldsymbol{\chi}}(t) = \begin{bmatrix} \dot{\mathbf{x}}^s(t) \\ \dot{\boldsymbol{\eta}}_a(t) \end{bmatrix} = \begin{bmatrix} \mathbf{f}\left(\mathbf{x}^s(t), \boldsymbol{\eta}, \mathbf{u}(t)\right) \\ \mathbf{f}_a\left(\mathbf{x}^s(t), \boldsymbol{\eta}, \mathbf{u}(t)\right) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}(t) \\ \boldsymbol{\omega}_a(t) \end{bmatrix}$$

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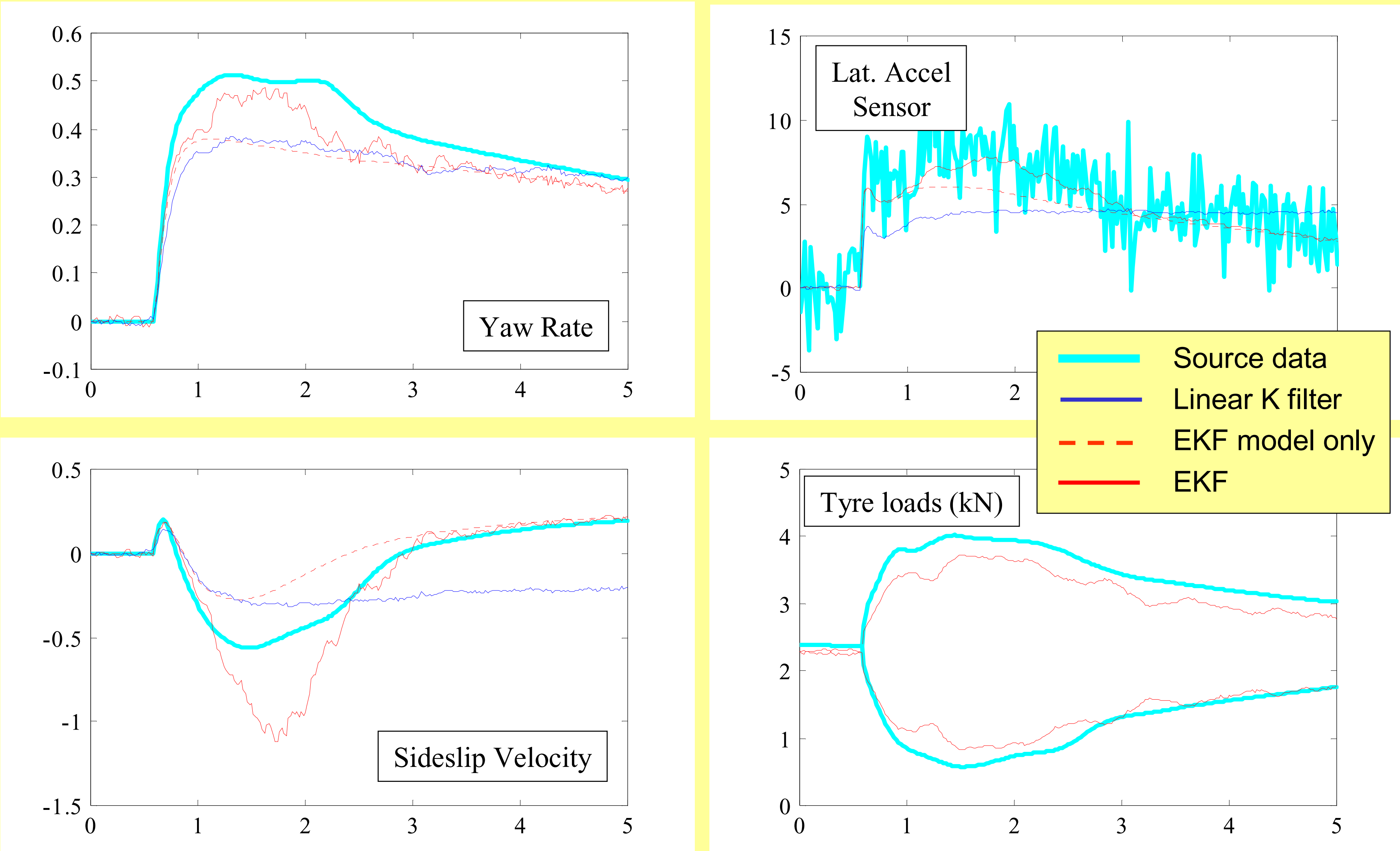
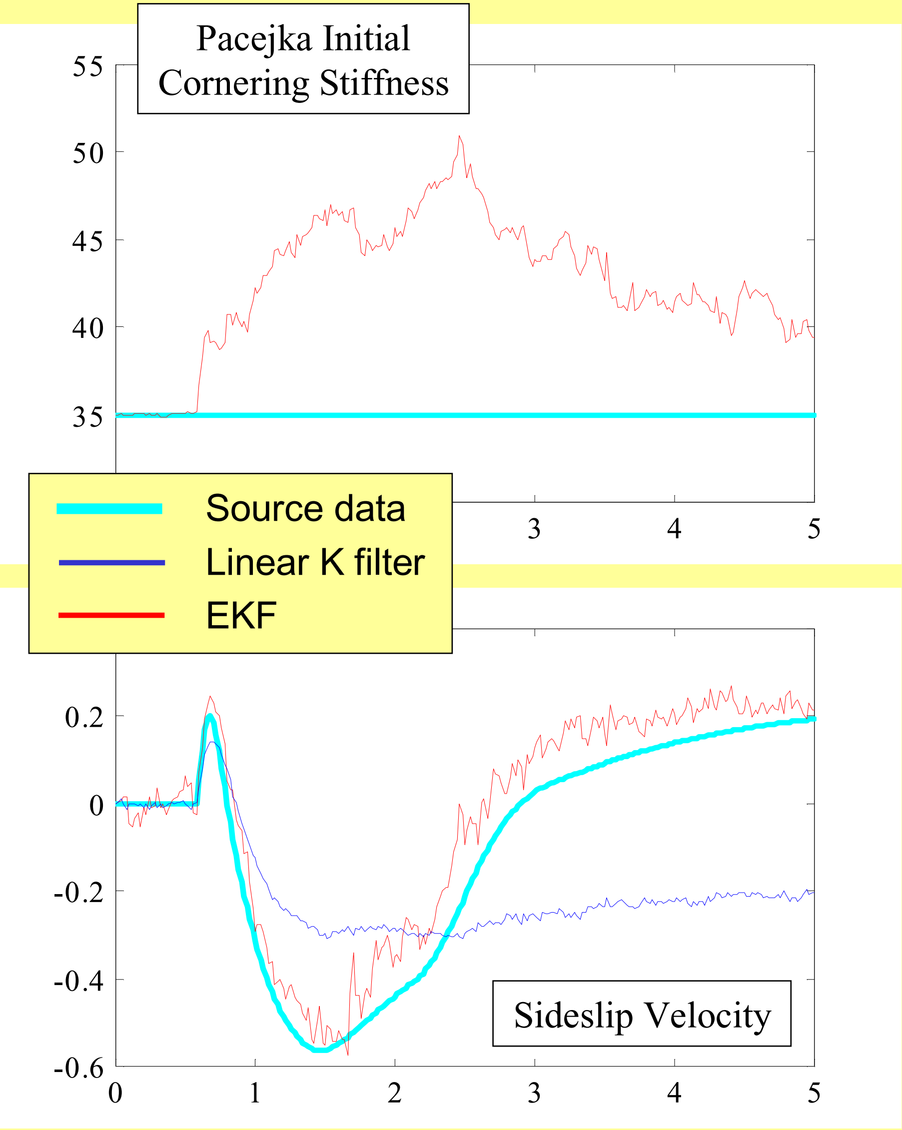
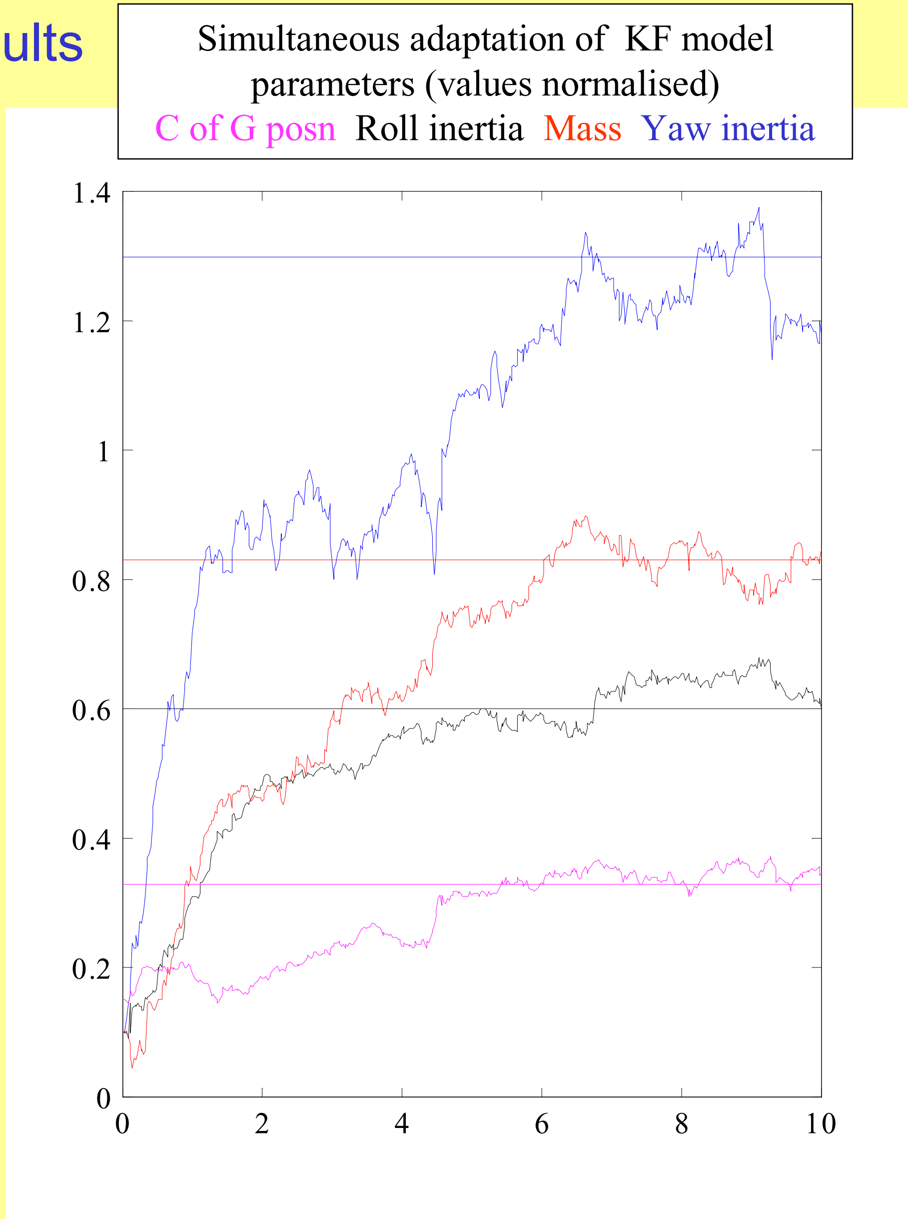
Study 3 results



Study 3 results





Conclusion : Sideslip Estimation

Accurate steady-state Sideslip velocity estimation is limited by model, and particularly tyre model accuracy